An Ontology of Shape and Growth of Structure

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The What and Why of Shape

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For physics of the whole universe, these references must be intrinsic, by definition of the universe. $\longrightarrow$ relations

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Leibniz's Principle of the Identity of Indiscernibles: In nature there cannot be two individual things that differ in number alone. For where there are two things it must be possible to explain why they are different. (First Truths, 1686)

$$\forall x \forall y \Big(\forall P \, (P(x) \leftrightarrow P(y)) \rightarrow x = y \Big)$$

If we consider points in space (e.g., $\mathbb{R}^3$), as in N-body model, $\mathcal{Q} = \mathbb{R}^{3N}$.

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See "Pure Shape Dynamics: Relational General Relativity" (arxiv: 2503.00996).

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Shape is scale-less.

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In Quantum Mechanics, there is representation problem (stemming from unitarity and the need for a preferred basis). It is addressed by $\mathcal{S}$.

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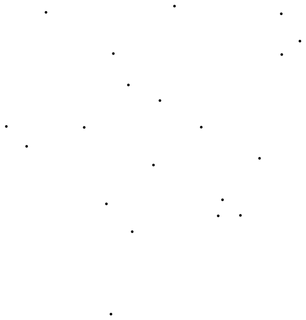
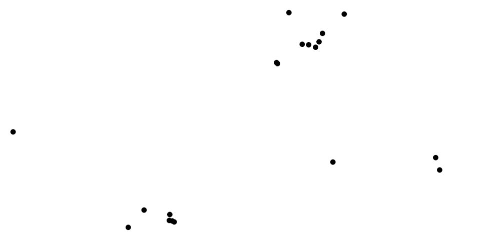
Dynamically, V_S generates attractor behavior in the evolution on $\mathcal{S}$
$\longrightarrow$ generation of typical dynamical arrows subject to V_S





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Principle of Monotonic Growth (PMI): The dynamics on $\mathcal{S}$ is such that the function C increases monotonically along dynamical trajectories.

$\longrightarrow$ The temporal structure supervenes on the functional C defined on shape space, including both duration and direction.
$\longrightarrow$ preferred internal shape clock

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Construction for N-body case:
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Model (Farokhi, Koslowski: unpublished)

$$\begin{aligned} \frac{dq^a}{ds} &= \frac{\alpha\, C^{,a}}{|\nabla C|^2} + A h^{ab} u_b, \\ \frac{du_a}{ds} &= -\frac{1}{2} g^{cd}_{,a} u_c u_d. \end{aligned}$$

$h_{ab} = g_{ab} - \frac{C_{,a} C_{,b}}{|\nabla C|^2}$

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Two proposals for dynamics satisfying the Principle of PMI.

First approach: C defines a preferred internal time.
$\longrightarrow$ foliation of shale space into hypersurfaces of constant C:

$$\mathcal{S} = \bigcup_C \mathcal{S}^C$$

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This equation can be viewed dynamically:

$$\Psi_f[\phi^{C_f}] = \mathcal{N} \int D[\phi] \Psi_i[\phi] \int_{\substack{\phi_i = \phi^{C_i} = \phi \\ \phi_f = \phi^{C_f}}} D[\phi^C] \, e^{i \int ds L^s[\phi^s]}$$

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More generally, one may introduce a superoperator $C[\rho]$ acting on the state.

This leads to a generalized (modified) von Neumann equation satisfying PMI:

$$\frac{d}{dC}\rho = -i[\hat{H}, \rho] + \mathcal{L}[C[\rho], \rho].$$

(Farokhi: *Implementation of PSD in QFT*, thesis)

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Illustration:

$$C = \langle\Psi| \int \frac{\hat{\phi}(x)\hat{\phi}(y)}{|x-y|} |\Psi\rangle$$

$$C = \text{const} \longrightarrow \Psi[\phi]\phi_k \ll \frac{1}{k} \quad \text{as } k \to \infty$$